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Approach to a Theory for the Masses of the Electron, Muon, and Heavier Charged Leptons

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The electron mass value $m = 9.093 \times 10^{-28}$ g is shown to be obtainable from the QED self-energy formula amended to include gravitational self-energy and restricted by the space–time equipartition law $m/m_0 = \frac{4}{3}$. Held together by gravity, the electron has a radius $r_e = 8\pi\hbar Gm/9e^2c = 2.588 \times 10^{-53}$ cm. An extension of the analysis yields the theoretical muon–electron mass ratio of 206.241 and predicts heavier charged leptons with mass values of 1.915 GeV, 15.67 GeV, and above.

Consider the hypothesis that the electron bare mass m_0 is a finite Lorentz-invariant scalar associated with static energy in the 3 spatial dimensions, while the experimental electron mass m contains an additional increment due to dynamics in 4-dimensional space–time. Then the symmetry of space–time suggests the equipartition law with respect to dimensions: $m/m_0 = \frac{4}{3}$. Indeed, the latter relation would arise if, for example, m_0 derives from a conserved effective stress-energy tensor which differs from the electrostatic Maxwell form by terms of order $\vec{v}/c$. Then for nonrelativistic velocities the electron momentum $m\vec{v} = \frac{4}{3}m_0\vec{v}$ is associated with the electrostatic field energy m_0c^2, as in the classical Abraham–Lorentz model (e.g., Møller, 1952). The following analysis shows that the experimental electron mass m can be derived with the aid of reasonable assumptions from the QED self-energy formula, amended to include gravitational self-energy and restricted by the space–time equipartition law $m/m_0 = \frac{4}{3}$. (For earlier approaches to this problem, see Rosen, 1971, and works cited therein.)

With physical units such that $c = 1$, we have (e.g., Feynman, 1961)

$$m = m_0 + \frac{3\alpha}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{\hbar K}{m_0}\right)\right]m_0 - \frac{Gm^2}{r} \tag{1.1}$$

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where m_0 is the bare mass, $\alpha \equiv e^2/\hbar = (137.036)^{-1}$, K is the invariant QED cutoff, $G = 6.673(\pm 0.003) \times 10^{-8}\ \mathrm{cm^3\ g^{-1}\ s^{-2}}$ is Newton's constant, and r is the invariant electron radius associated with the gravitational self-energy. On physical grounds, it is plausible to fix the cutoff in (1.1) by setting $K \equiv m_0/2mr$, for then the argument of the logarithm is the geometric-optical ratio of the electron Compton wavelength to the electron diameter $2r$. Assigning the latter physical value to K and eliminating $m_0 = 3m/4$, we obtain

$$\frac{1}{3} = \frac{3\alpha}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{\hbar}{2mr}\right)\right] - \frac{4Gm}{3r} \tag{1.2}$$

The physical radius is assumed to maximize the right side of (1.2),[1] and by equating the derivative with respect to r to zero we find

$$r = 8\pi Gm/9\alpha \tag{1.3}$$

Putting the radius formula (1.3) into (1.2) and solving the resulting equation yields

$$m = m_e = \left(\frac{9e^2}{16\pi G}\right)^{1/2}\left[\exp - \left(\frac{\pi}{9\alpha} + \frac{3}{8}\right)\right]$$

$$= \frac{|e|}{G^{1/2}}(4.89025 \times 10^{-22}) = 9.093 \times 10^{-28}\ \mathrm{g} \tag{1.4}$$

which is only about 0.175% less than the experimental value (inaccuracy about eight times the percentile uncertainty in the current experimental value for $G^{1/2}$). Since our electron radius (1.3) is about 200 times greater than Schwarzschild's, we have a posteriori justification for employing the Newtonian self-energy in (1.1). The very small magnitude of the radius $r = 2.588 \times 10^{-53}$ cm means that the electron will appear to be point-like in foreseeable experiments.

We extend this theoretical framework to include the muon and heavier charged lepton states (CLS) by introducing a principal quantum number label: $n = 0$ for the electron (ground CLS), $n = 1$ for the μ (first excited CLS), $n = 2$ for the τ (second excited CLS), etc. On the complete Hilbert space spanned by the CLS, diagonal and positive self-adjoint operators are associated with m and m_0, the ratio m/m_0 decreasing monotonically with increasing n from the equipartition value of $\frac{4}{3}$ for $n = 0$ and tending asymptotically to unity as $n \to \infty$. From this physical assumption of "quasi-equi-

[1] An extremal condition on an energy function that depends on r can be employed to make an analogous derivation of the Bohr radius (Feynman, 1965). The rigorous basis for this in atomic quantum theory (and probably also in the present context) is an underlying Rayleigh–Ritz procedure for stationary states in which r enters as a variational parameter.

TABLE 1. Theoretical mass and radius values for the charged leptons given by (1.8) and (1.7).

n	0	1	2	3	4	∞
m/m_e	1	206.241	3748.13	3.0662×10^{4}	1.6128×10^{5}	6.8670×10^{20}
m(MeV)[a]	0.5110034	105.390	1915.31	1.5668×10^{4}	8.2415×10^{4}	3.5093×10^{20}
m(MeV)[b]	0.51008	105.2	1912	1.564×10^{4}	8.227×10^{4}	3.503×10^{20}
r(cm)	2.588×10^{-53}	5.189×10^{-51}	9.284×10^{-50}	7.508×10^{-49}	3.913×10^{-48}	1.333×10^{-32}

[a] With the current experimental value for m_e.
[b] With the theoretical value (1.4).

partition" for excited states, it follows that the operator associated with $[4 - (3m/m_0)] \equiv \sin^4 \theta_n$ is self-adjoint, diagonal, and has its spectrum on the unit interval; moreover, the so-defined angular variable θ_n has quantized values such that $\theta_0 = 0 < \theta_1 < \cdots < \theta_n < \theta_{n+1} \cdots < \theta_\infty = \pi/2$. Consistent with this picture of quasi-equipartition, we conjecture the CLS quantum condition $4 \tan^4 \theta_n = n$, where the prefactor of four and the quartic power are to be derived in a future theory. By employing the quantum condition in the definition above for $\sin^4 \theta_n$, we have

$$m/m_0 = \tfrac{1}{3}[4 - n(n^{1/2} + 2)^{-2}] \tag{1.5}$$

and in place of (1.2) we obtain

$$\tfrac{1}{3}[1 - n(n^{1/2} + 2)^{-2}] = \frac{3\alpha}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{\hbar}{2mr}\right)\right] - \frac{Gm}{3r}[4 - n(n^{1/2} + 2)^{-2}] \tag{1.6}$$

The right side of (1.6) is a maximum for

$$r = 2\pi[4 - n(n^{1/2} + 2)^{-2}]Gm/9\alpha \tag{1.7}$$

Substituting the radius formula (1.7) into (1.6) yields

$$m = m_e[1 - \tfrac{1}{4}n(n^{1/2} + 2)^{-2}]^{-1/2} \exp\,[\pi n/9\alpha(n^{1/2} + 2)^2] \tag{1.8}$$

in which m_e is given by (1.4). As shown in Table 1, the m/m_e values generated by (1.8) are wholly consistent with experiment, while giving rise to values of the radius (1.7) which make the charged leptons effectively point-like. In particular, the muon–electron mass ratio of 206.241 obtained from (1.8) for $n = 1$ is only 0.25% less than the current experimental value, the $n = 2$ tau (Perl, 1976, known by experiment to lie in the range 1.8–2.0 GeV) is pin-pointed at 1.915 GeV, and the $n = 3$ charged lepton mass is given as 15.67 GeV. The latter predictions afford a way of checking the essential correctness of the formulas with future experiments.

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